

THE PERMEABILITY OF POROUS MEDIA TO LIQUIDS AND GASES†

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ABSTRACT

The standard procedure for determining the permeability of porous media according to *API Code No. 27* (first edition, October 1935) is based on the fundamental assumption that, as long as the rate of flow is proportional to the pressure gradient, the permeability constant of a porous medium is a property of the medium, and is independent of the fluid used in its determination.

Although this is true for most liquids, the permea-

bility constant as determined with gases is dependent upon the nature of the gas, and is approximately a linear function of the reciprocal mean pressure. This effect can be explained by taking into account the phenomena of slip, which are related closely to the mean free paths of the gas molecules. The apparent permeability extrapolated to infinite pressure gives a permeability constant which is a characteristic of the porous medium only.

I. Introduction

It has become common practice in the oil industry to determine the permeability of core material with dry air; the equipment usually employed for this determination is arranged to operate with the outlet of the sample at or near atmospheric pressure.⁴

This practice is based on the fundamental assumption that, as long as Darcy's law is obeyed, i.e., as long as the rate of flow is proportional to the pressure gradient,[†] the permeability constant of a porous medium is a property of the medium, and is independent of the fluid used in its determination.⁵ Therefore, the results obtained by laboratory measurements with air are taken to be applicable to the homogeneous flow of either oil or gas in underground reservoirs.

The fundamental assumption that the permeability of a porous medium is independent of the fluid used in its determination is illustrated by Muskat⁶ with a few measurements of highly permeable sandstones to air and liquid.

However, Muskat⁶ gives a table of results of measurements on the permeability to water and air of a number of oil sands, carried out by Fancher, Lewis, and Barnes,⁷ showing large discrepancies between the permeability to air and water—most values found for water being lower than for air. Many cases of such discrepancies between the permeabilities to air and those to water and other liquids also were observed during investigations carried out in the laboratories of

the Bataafsche Petroleum Maatschappij, Amsterdam, The Netherlands, and of the Shell Development Company, Emeryville, Calif. In general it was found that, with highly permeable media, the differences between liquid and air permeabilities were small, whereas these differences were considerable for media of low permeability.

These discrepancies made it desirable to investigate the validity of the assumption that the permeability of a porous medium is independent of the nature of the fluid with which the determination is carried out.

The investigation has shown that the permeability to a gas is a function of the mean free path of the gas molecules, and thus depends on factors which influence the mean free path, such as the pressure, temperature, and the nature of the gas. Therefore, when the mean free paths are small, e.g., at high pressures, the permeability to gas should be expected to approach that for liquids. The experimental data which support these conclusions are discussed hereinafter.

II. Permeability of Porous Media to Different Liquids

It is, of course, obvious that if a liquid reacts with some constituent of the core material, e.g., if water causes clay-containing core material to swell, then differences between the permeabilities for different liquids and air can be expected. The same is true if the core material is poorly consolidated so that part of the pores may be plugged off by loose material eroded by the liquid. From a practical point of view, these considerations may be very important; thus, if a problem arises about the water movement in clay-containing formations, the permeability of the formation to dry

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† Presented by A. G. Loomis, Shell Development Co., Emeryville, Calif., at Eleventh Mid-Year Meeting, Tulsa, Okla., May 1941.

[‡] Muskat⁶ has thoroughly discussed the limits of validity of Darcy's law with respect to the transition of laminar into turbulent flow. In the present study the experimental conditions were chosen in the region of laminar flow.

⁴ Figures refer to REFERENCES on p. 211.

air might be of no significance or even definitely misleading.

The question to be discussed here, however, is whether porous media in which no changes in the internal structure take place will show different permeability constants to different fluids.

It has been a point of much discussion in the literature⁷ whether the walls of small capillaries, as they occur in porous media, are able to adsorb molecules to such an extent as to build up more or less rigid layers several molecules thick. Such adsorbed layers would reduce the effective pore diameter to a different degree for different liquids and, as a consequence, the permeability of a porous mass would depend on the nature of the liquid. Some authors⁸ prefer to speak about an increase in viscosity of the liquid with a decreasing distance to the solid wall—which amounts, of course, to the same thing.

TABLE 1

Permeability Constant
(Millidarcys)

Liquid Used	G ₁ Filter No. 1	G ₁ Filter No. 2	G ₁ Filter C	G ₁ Filter E
Kerosine	48.6	49.8
Toluene	47.6	50.2	3.70	3.42
Chloroform	49.8	50.6	3.66	3.36
Diethyl ether	50.7	51.9
Carbon tetrachloride...	49.5	50.8	3.68	3.38
Pentane	50.2	50.1
Isooctane	3.67	3.42
Nitrobenzene	3.27	3.05

A careful survey of the literature, however, leads to the conclusion that for flow through capillaries there is no evidence for the presence of adsorption layers of such a thickness as to decrease perceptibly the liquid permeability of a porous medium.

In order to test this experimentally for porous media and to avoid complications by swelling or erosion of such material, permeability measurements were carried out on Jena glass filters.

The results given in Table 1 were obtained.

The deviations, with the exception of those for nitrobenzene, are within the experimental error.

Water behaves normally, as may be seen from Table 2.

To verify further the deviation found for nitrobenzene, permeability measurements were carried out on other filters; first the permeability to isooctane was determined, then to nitrobenzene and, finally, the permeability to isooctane was determined again. The results are given in Table 3.

The deviations shown by nitrobenzene, however, are not a result of adsorption layers on the walls of the capillaries, but are caused by electro-kinetic phenomena. When a liquid flows through a capillary, a difference in potential is formed between the ends of the capillary:

the streaming potential. This streaming potential gives rise to an electro-endosmotic liquid flow, which is opposite to the hydrodynamic flow. A similar phenomenon takes place in a porous medium. The permeability found, consequently, is too low, dependent on the nature of the liquid. The characteristics of most liquids (i.e., the value of zeta potential, dielectric constant, and specific conductivity) are such as to cause only small electro-endosmotic effects.

In the case of nitrobenzene the electro-endosmotic effect could be made vanishingly small by increasing the conductivity (simultaneously decreasing the zeta potential) of the nitrobenzene by dissolving a very small

TABLE 2

Permeability Constant
(Millidarcys)

Liquid Used	G ₁ Filter	G ₁ Filter
Isooctane	78.8	2.31
Water	79.4	2.27

TABLE 3

Permeability (Millidarcys)

Liquid Used	G ₁ Filter A	G ₁ Filter B	G ₁ Filter
Isooctane	2.38	2.26	63.8
Nitrobenzene	2.17	2.01	62.8
Isooctane	2.34	2.24	...

TABLE 4

Nature
of the Gas

Permeability
(Millidarcys)

Air	3.06
Hydrogen	4.23
Carbon dioxide	2.88

amount of a ferric salt in the liquid. In doing so a permeability was found which was the same within the experimental error as for other liquids such as isooctane.

Therefore, it can be concluded that, if no changes in the internal structure of the porous medium can take place, the differences in permeability to different liquids are in most cases within the experimental error. The discrepancies found between air permeability and liquid permeability [as discussed under (I)] are apparently due not to the nature of the liquids used in the determination, but to the value of the air permeability following the ordinary procedure. That the discrepancies were not due to the behavior of the liquids, but to the behavior of the gas, became clear at once when the permeability of a Jena glass filter was measured with different gases, as shown in Table 4.

The explanation of the differences between the

permeabilities to different gases is found by applying the results for the flow of *dilute* gases, as found by Kundt and Warburg¹² and by Knudsen,¹⁴ to the flow of gases at *normal pressure* through porous media.

III. Theory of Slip

It has been proved by Kundt and Warburg¹² that, when a gas is flowing along a solid wall, the layer of gas next to the surface is in motion with respect to the solid surface. In other words, if the wall has a zero velocity, then the velocity of the gas layer in the immediate vicinity of the wall has a finite value. As a consequence the quantity of gas flowing through a capillary is larger than would be expected from Poiseuille's formula. This becomes evident by the following simplified considerations of the kinetic theory of gases.

Consider a layer adjacent to the wall which is thinner than the mean free path λ of the gas molecules, so that practically a molecule does not collide with other molecules present in this layer. At a given moment half of the gas molecules in this layer will have a component of velocity moving towards the wall; the other half in the opposite direction. The molecules moving towards the wall have had their last collision somewhere in the flowing mass, and, therefore, will have an average velocity component in the direction of flow different from zero. A part of this average velocity component will be lost in colliding with the wall. Even if the molecules lose it entirely, then still the average velocity component in the direction of flow of all the molecules contained in the layer will amount to half of the average velocity component of the molecules moving towards the wall. The gas in the layer, therefore, will have a finite rate of flow. Let w_0 be the velocity near the wall with respect to this wall. Assuming a constant velocity gradient in the direction perpendicular to the wall $\frac{dw}{dz}$, the velocity at a distance z from the wall will be:

$$W = w_0 + z \frac{dw}{dz}$$

We suppose, furthermore, that the molecules which had their last collision at a distance z from the wall have an average velocity component in the direction of flow equal to the velocity of flow of the gas in this place. The average distance z from the wall at which the last collision took place will be proportional to the mean free path λ of the gas molecules $z = c\lambda$, in which c is a proportionality factor.

Hence the average velocity component of the molecules moving towards the wall is:

$$w_0 + c\lambda \frac{dw}{dz}$$

If the collision with the wall is entirely inelastic, so that all the molecules lose their average velocity component in the direction of flow, then, as stated above, the average velocity of all the molecules contained in the layer under consideration will amount to the half

of the average velocity component of the molecules moving towards the wall.

Hence:

$$\begin{aligned} w_0 &= \frac{1}{2} \left(w_0 + c\lambda \frac{dw}{dz} \right) \\ w_0 &= c\lambda \frac{dw}{dz} \end{aligned} \quad (1)$$

Kundt and Warburg showed that the assumption that the velocity gradient $\frac{dw}{dz}$ is constant does not hold; near the wall the velocity gradient is greater than at a large distance from the wall. Furthermore, in the above considerations it has been assumed that the molecules after having collided in a certain region have an average velocity component in the direction of flow equal to the velocity of flow in this region. The average velocity gradient, however, depends on the velocity-distribution function in this region; hence, not only on the velocity, but also on the velocity gradient, as was pointed out by Jäger¹⁵ and Jeans.¹⁶ The value of c , as calculated by Kundt and Warburg, therefore, is inaccurate. It has not yet been possible to make an exact calculation, but the value of c seems to be slightly less than 1.

IV. Flow of Gas through a Straight Capillary

Warburg¹² applied the effect of slip to the flow of gas through a capillary whose radius is large compared with the mean free path. Following Warburg, consider a capillary of radius r_0 and length l with its axis coincident with the x -axis, and assume that the velocity is a function only of the distance r from the axis. The differences of the drags in the x -direction on the two sides of a cylindrical shell of gas, whose radii are r and $r+dr$ contained between the cross-sections x and $x+dx$, give rise to a force in a direction opposite to the flow:

$$- \frac{d}{dr} \left(\mu \frac{dw}{dr} \cdot 2\pi r dx \right) dr$$

In which μ is the viscosity of the gas.

For a stationary motion, this force must be balanced by the normal pressures on the plane ends of the shell. The difference of these normal pressures is:

$$- \frac{dp}{dx} dx \cdot 2\pi r dx$$

Hence:

$$\begin{aligned} - \frac{d}{dr} \left(\mu \frac{dw}{dr} \cdot 2\pi r dx \right) dr &= - \frac{dp}{dx} dx \cdot 2\pi r dx \\ \frac{d}{dr} \left(r \frac{dw}{dr} \right) &= \frac{dp}{dx} \cdot \frac{r}{\mu} \end{aligned}$$

Integration gives:

$$r \frac{dw}{dr} = \frac{1}{2\mu} \cdot \frac{dp}{dx} r^2 + A \quad (2)$$

Integrating once more:

$$w = \frac{1}{4\mu} \cdot \frac{dp}{dx} r^2 + A \ln r + B \quad (3)$$

As the velocity must be finite at the axis ($r=0$), $A=0$. The integration constant B can be found by putting in

the boundary condition [equation (1)] at the wall ($r=r_0$).

$$w_0 = c\lambda \frac{dw}{dz} = -c\lambda \left(\frac{dw}{dr} \right)_{r=r_0}$$

Substituting this in equation (3):

$$-c\lambda \left(\frac{dw}{dr} \right)_{r=r_0} = \frac{1}{4\mu} \frac{dp}{dx} r_0^2 + B$$

$$B = -c\lambda \left(\frac{dw}{dr} \right)_{r=r_0} - \frac{1}{4\mu} \frac{dp}{dx} r_0^2$$

According to equation (2):

$$\left(\frac{dw}{dr} \right)_{r=r_0} = \frac{r_0}{2\mu} \frac{dp}{dx}$$

$$B = -c\lambda \frac{r_0}{2\mu} \frac{dp}{dx} - \frac{1}{4\mu} \frac{dp}{dx} r_0^2$$

This substituted in equation (3):

$$w = -\frac{1}{4\mu} \frac{dp}{dx} (r_0^2 - r^2 + 2c\lambda r_0)$$

The volume passing during the time t through a shell between two cylinders of radii r and $r+dr$ is:

$$dv = -\frac{\pi}{2\mu} \frac{dp}{dx} \cdot t (r_0^2 r dr - r^3 dr + 2c\lambda r_0 r dr)$$

The volume passing through the whole cross-section is:

$$v = -\frac{\pi}{2\mu} \frac{dp}{dx} t \int_0^{r_0} (r_0^2 r dr - r^3 dr + 2c\lambda r_0 r dr)$$

$$= -\frac{\pi r_0^4}{8\mu} \frac{dp}{dx} \cdot t \left(1 + \frac{4c\lambda}{r_0} \right) \quad (4)$$

As the mean free path is inversely proportional to the pressure, we may write:

$$4c\lambda = \frac{b}{p}$$

If per-unit time n gram moles pass through the capillary, then:

$$\frac{pv}{t} = nRT$$

This substituted in equation (4):

$$nRT dx = -\frac{\pi r_0^4}{8\mu} \left(p dp + \frac{b}{r_0} dp \right)$$

Integrating this over the whole length of the capillary:

$$nRT l = -\frac{\pi r_0^4}{8\mu} \int_{p_1}^{p_2} \left(p dp + \frac{b}{r_0} dp \right) = \frac{\pi r_0^4}{8\mu} \left\{ \frac{p_1^2 - p_2^2}{2} + \frac{b}{r_0} (p_1 - p_2) \right\}$$

Therefore:

$$\frac{pv}{t} = Q = \frac{\pi r_0^4}{8\mu l} \cdot (p_1 - p_2) \cdot \frac{p_1 + p_2}{2} \cdot \left(1 + \frac{b}{\frac{1}{2}(p_1 + p_2)r_0} \right)$$

If $\bar{\lambda}$ is the mean free path at the mean pressure $\bar{p} = \frac{1}{2}(p_1 + p_2)$, then:

$$Q = \frac{\pi r_0^4}{8\mu l} \cdot (p_1 - p_2) \cdot \bar{p} \cdot \left(1 + \frac{4c\bar{\lambda}}{r_0} \right) \quad (5)$$

If there is no slipping of the fluid in contact with the wall ($c\bar{\lambda} = 0$) then equation (5) is reduced to Poiseuille's equation.

V. Flow of a Gas through an Idealized Porous Medium

The simplest picture which can be formed of the

laminar flow of fluids through a porous medium is that in which all the capillaries in the material are of the same diameter and are oriented at random through the solid material. Such a model, although it would have the same permeability in all directions, is, of course, in many respects too simple to use as a basis for quantitative predictions; but, as shown herein-after, it makes possible qualitative predictions.

Consider a cube of the material with an edge of 1 cm; the direction of flow is perpendicular to one of the planes of the cube; and let there be n capillaries of radius r .

The amount of liquid flowing through per unit time is found by applying Poiseuille's law:

$$\frac{v}{t} = \frac{1}{8} \cdot \frac{n\pi r^4}{\mu} (p_1 - p_2) \quad (6)$$

According to Darcy's law, the amount of liquid passing through would be given by:

$$\frac{v}{t} = \frac{K}{\mu} \cdot (p_1 - p_2) \quad (7)$$

in which K is the permeability constant.

Combining equations (6) and (7) gives:

$$K = \frac{1}{8} \cdot \frac{n\pi r^4}{\mu} \quad (8)$$

For a gas, if slipping of the gas in contact with the wall is taken into account [see equation (5)]:

$$Q = \frac{1}{8} \cdot \frac{n\pi r^4}{\mu} \cdot (p_1 - p_2) \bar{p} \left(1 + \frac{4c\bar{\lambda}}{r} \right) \quad (9)$$

Or, combined with equation (8):

$$Q = \frac{K}{\mu} \cdot (p_1 - p_2) \bar{p} \left(1 + \frac{4c\bar{\lambda}}{r} \right) \quad (10)$$

Darcy's law gives, for the flow of gas through a porous medium of the above dimensions:

$$Q = \frac{K_a}{\mu} \cdot (p_1 - p_2) \bar{p} \quad (11)$$

Equations (10) and (11) lead to:

$$K_a = K \left(1 + \frac{4c\bar{\lambda}}{r} \right) \quad (12)$$

As the mean free path is inversely proportional to the pressure, we may write:

$$\frac{4c\bar{\lambda}}{r} = \frac{b}{\bar{p}}$$

in which b is a constant. This substituted in equation (12) gives:

$$K_a = K \left(1 + \frac{b}{\bar{p}} \right) \quad (13)$$

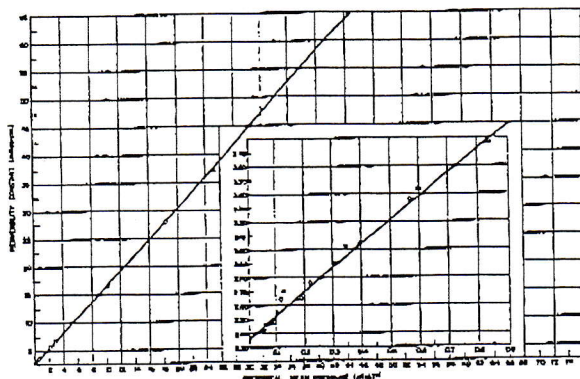
a relation between the apparent and true permeability of an idealized porous system to gas.

VI. Permeability of Core Samples and Glass Filters to Various Gases at Different Pressures

If the simplified considerations given in Sect. V are valid not only for a system of straight capillaries, but

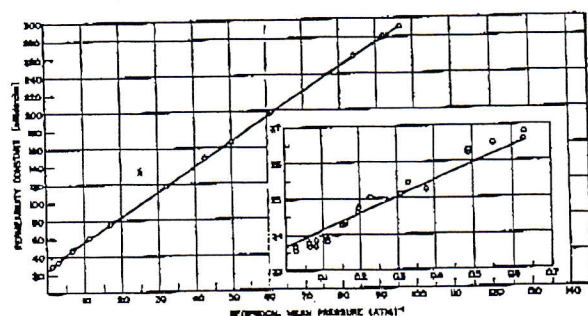
can be applied to actual porous media, then, according to equation (12) or (13), it would follow that:

1. Gas permeability is a linear function of the reciprocal mean pressure.
2. Gas permeability does not depend on the pressure difference ($p_1 - p_2$) as long as the mean pressure $\bar{p} = \frac{p_1 + p_2}{2}$ is constant.
3. As the constant b is inversely proportional to r (the radius of the capillaries) the value of b may be expected to be small for highly permeable samples, and to be larger for less permeable samples.



Permeability Constant of Jens Glass Filter G4A to Air at Different Pressures (Permeability Constant to Isooctane, 2.36 Md).

FIG. 1



Permeability Constant of Core Sample "A" to Air at Different Pressures (Permeability Constant to Isooctane, 23.66 Md).

FIG. 2

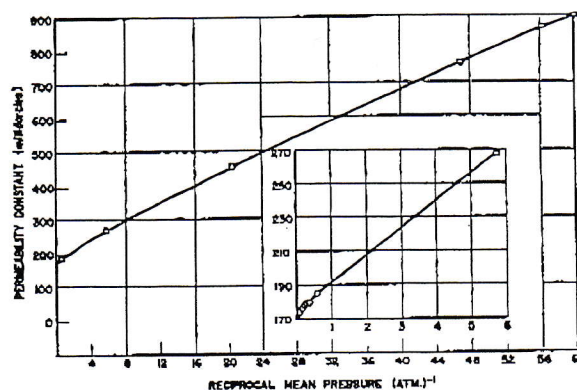
4. At the same mean pressure, the permeability is different for different gases, as the mean free path λ has different values in this case.
5. The apparent permeability K , extrapolated to infinite pressure ($\frac{1}{p} = 0$) should give the true permeability.

To verify the above conclusions, permeability measurements have been made with glass filters and with core samples from a large variety of fields.

In order to test the first conclusion, i.e., that the gas permeability is a linear function of the reciprocal of the mean pressure, gas-permeability measurements were carried out at different pressures, from about 1 cm mercury up to 20 atmospheres. In Table 5 results are given of measurements of a glass filter to air at various pressures; in Tables 6 and 7 similar measurements are given for two core samples.

Fig. 1, 2, and 3 show that the apparent permeability is approximately a linear function of the reciprocal mean pressure. This linear function, however, is an approximation, as becomes evident from Tables 5, 6, and 7, wherein the value of constant b increases with increasing pressure.

Even with an idealized pore system, the factor b cannot be expected to be constant, as the theory of Kundt and Warburg cannot be applied to the flow of gas through a capillary if the mean free path is no longer small compared with the radius of the capillary (i.e., deviations to be expected at reduced pressures).



Permeability Constant of Core Sample "F" to Air at Different Pressures (Permeability Constant to Isooctane, 170 Md).

FIG. 3

In the second place, it has been assumed that a law of flow is applicable which in fact is strictly valid only for capillaries of such a length that the influence of the ends is negligible. The capillaries in a porous system, however, will not have a constant cross-section over a length larger than their diameter.

This change in the factor b , however, will not be discussed here in detail.

As to the second conclusion, i.e., that the permeability does not depend on the pressure difference ($p_1 - p_2$) as long as the mean pressure is constant, this has been verified several times and found always to be the case. The data in Tables 5 and 6 illustrate this point.

From the examples already presented and from others, shown in Table 8, it is evident that the factor b is smaller the higher the permeability of the sample, as may be expected.

To verify the conclusion that at the same mean pressure the permeability is different for different gases,

measurements were carried out with air, nitrogen, carbon dioxide, and hydrogen. The results are shown in Fig. 4, 5, 6, and 7. According to equation (12), at a given pressure the apparent permeabilities for different gases should increase in the order of their mean free paths, which is confirmed by the present data. The same permeability for different gases, however, should be found if the permeabilities are compared at pressures when the mean free paths of the gas molecules are the

TABLE 5

Permeability Constant of Jena Glass Filter "G. A" to Air at Different Pressures

(Permeability Constant to Isooctane: 2.36 Md)

$\bar{P} = \frac{P_1 + P_2}{2}$ (Atmospheres)	$P_1 - P_2$ (Atmospheres)	$\frac{1}{P}$ (Atmospheres) ⁻¹	K_a (Millidarcys)	$\frac{K_a}{K}$	b
0.02195	0.01771	45.55	64.84	27.4	0.58
0.04007	0.01694	24.96	37.13	15.7	0.59
0.05505	0.01591	18.18	27.96	11.8	0.60
0.09810	0.01508	10.19	16.65	7.05	0.59
0.34976	0.00952	2.859	6.69	2.84	0.64
0.4615	0.00757	2.167	5.64	2.39	0.64
1.192	0.0450	0.888	3.76	1.59	0.71
1.199	0.0343	0.834	3.76	1.59	0.71
1.6679	0.0546	0.600	3.43	1.45	0.75
1.683	0.0270	0.594	3.43	1.45	0.76
1.758	0.0462	0.569	3.36	1.42	0.75
2.517	0.04418	0.3974	3.04	1.29	0.72
2.908	0.0358	0.3439	3.02	1.28	0.81
3.245	0.0445	0.3082	2.90	1.23	0.74
3.859	0.0422	0.2592	2.79	1.18	0.71
4.571	0.0430	0.2189	2.76	1.17	0.76
5.353	0.0458	0.1868	2.65	1.12	0.67
5.501	0.0318	0.1819	2.65	1.12	0.68
5.612	0.0481	0.1782	2.70	1.15	0.82
6.014	0.0294	0.1662	2.64	1.12	0.72
6.484	0.0419	0.1542	2.64	1.12	0.78
9.484	0.0329	0.1054	2.54	1.08	*
10.91	0.0289	0.0917	2.48	1.05	...
12.08	0.0450	0.0828	2.48	1.05	...
14.04	0.0335	0.0712	2.47	1.05	...
16.49	0.0282	0.0606	2.46	1.04	...
18.98	0.0185	0.0527	2.44	1.03	...
19.31	0.0210	0.0518	2.42	1.02	...

* At higher pressures the calculation of b has not been carried out, as the influence of the experimental error on the value of b is so great as to make that figure unreliable.

same. As the mean free path is inversely proportional to the pressure, the same permeability for different gases will be found at pressures which are proportional to the mean free paths. For hydrogen, air, and carbon dioxide, for instance, the mean free paths of which are, respectively, 1.123×10^{-6} cm, 608×10^{-6} cm, and 397×10^{-6} cm, the same permeability should be found at pressures in the ratio 1.85:1:0.65, or whose reciprocals are in the ratio of 0.54:1:1.54. As may be seen, the experimental results check this prediction quite closely.

Finally (as may be seen from Fig. 1, 2, 3, 5, 6, and 7)

the apparent permeability K_a extrapolated to infinite pressure is, within the experimental error, equal to the liquid permeability. This is illustrated further in Table 9, in which the permeabilities at atmospheric pressure, the extrapolated permeabilities at infinite pressure, and the liquid permeabilities of a number of

TABLE 6

Permeability Constant to Core Sample "A" to Air at Different Pressures

(Permeability Constant to Liquid: 23.66 Md)

$\bar{P} = \frac{P_1 + P_2}{2}$ (Atmospheres)	$P_1 - P_2$ (Atmospheres)	$\frac{1}{P}$ (Atmospheres) ⁻¹	K_a (Millidarcys)	$\frac{K_a}{K}$	b
0.01036	0.01627	96.56	290	12.25	0.116
0.01087	0.01603	92.00	280	11.82	0.117
0.01189	0.01577	84.09	259	10.96	0.118
0.01633	0.01508	61.20	197	8.31	0.119
0.02023	0.01425	50.05	165	6.98	0.120
0.02336	0.01414	42.83	148	6.25	0.122
0.03149	0.05734	31.74	117	4.97	0.124
0.05921	0.01081	16.89	75.6	3.20	0.130
0.08882	0.03962	11.26	60.0	2.53	0.136
0.1464	0.03101	6.829	46.6	1.97	0.142
0.3433	0.01635	2.913	34.2	1.446	0.153
0.799	0.00351	1.251	28.6	1.207	0.166
1.570	0.03652	0.636	26.79	1.133	0.209
1.581	0.01558	0.632	26.58	1.124	0.198
1.802	0.05379	0.5553	26.49	1.119	0.214
2.037	0.03422	0.4910	26.23	1.108	0.220
2.045	0.02052	0.4892	26.33	1.112	0.229
2.657	0.03511	0.3764	25.22	1.066	0.18
3.007	0.05166	0.3325	25.43	1.075	0.23
3.224	0.03704	0.3101	25.09	1.060	0.20
4.366	0.04800	0.2290	25.02	1.058	0.26
5.049	0.03296	0.1981	24.72	1.045	0.23
5.100	0.03493	0.1961	24.62	1.041	0.21
6.34	0.01779	0.1577	24.34	1.029	*
6.34	0.03729	0.1577	24.29	1.027	...
6.43	0.03446	0.1553	24.25	1.023	...
8.94	0.01615	0.1119	23.79	1.005	...
8.94	0.03177	0.1119	23.85	1.008	...
9.19	0.03221	0.1088	23.93	1.012	...
12.45	0.01707	0.0802	23.66	1.000	...
12.48	0.02033	0.0800	23.82	1.007	...
16.44	0.04056	0.0608	23.67	1.000	...
16.45	0.01954	0.0608	23.77	1.005	...
20.01	0.01821	0.0499	23.55	0.995	...
20.01	0.01105	0.0499	23.65	1.000	...

* At higher pressures the calculation of b has not been carried out, as the influence of the experimental error on the value of b is so great as to make that figure unreliable.

glass filters and core samples of different origin are given. That the gas permeability extrapolated to infinite pressure is equal to the liquid permeability is plausible, as the mean free path in the liquid state would be approached by that of gases at very high pressure.

It is evident from the foregoing results that the per-

TABLE 7

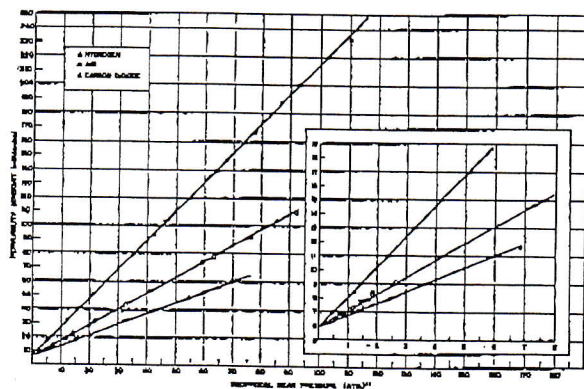
Permeability Constant of Core Sample "F" to Air at Different Pressures

(Permeability Constant to Liquid; 170 Md)

$\frac{P_1 - P_2}{P_1 + P_2}$ (Atmospheres)	$\frac{P_1 - P_2}{P_1 + P_2}$ (Atmospheres)	$\frac{1}{P}$ (Atmospheres) ⁻¹	K_a (Millidarcys)	$\frac{K_a}{K}$	b
0.01665	0.02717	60.1	900	5.29	0.071
0.01779	0.02652	56.2	863	5.07	0.072
0.02130	0.02528	46.96	758	4.44	0.073
0.04890	0.01821	20.46	455	2.67	0.082
0.1738	0.00843	5.75	268	1.58	0.082
1.701	0.02017	0.587	184.9	1.087	0.15
2.659	0.01625	0.3761	180.0	1.058	0.16
3.641	0.01511	0.2748	179.8	1.057	0.21
4.619	0.01412	0.2165	179.1	1.052	0.24
7.04	0.00821	0.1420	176.7	1.038	0.27
12.89	0.00582	0.0776	173.9	1.022	0.29

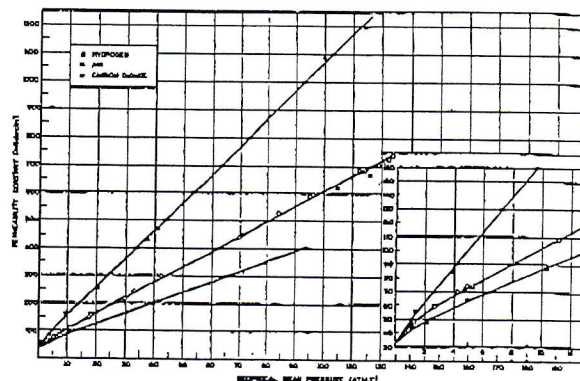
TABLE 8

Sample	Permeability for Isooctane (Millidarcys)	Factor b for $\frac{1}{P} = \infty$
Jena glass filter G, A	2.36	0.58
Jena glass filter G, No. 3...	2.13	0.76
Jena glass filter G, No. 5...	4.36	0.76
Jena glass filter G,	85.6	0.138
Core sample A	23.66	0.116
Core sample B	24.40	0.096
Core sample C	32.1	0.174
Core sample D	92.3	0.067
Core sample E	105	0.067
Core sample F	170	0.069
Core sample G	1,054	0.035
Core sample H	1,347	0.022



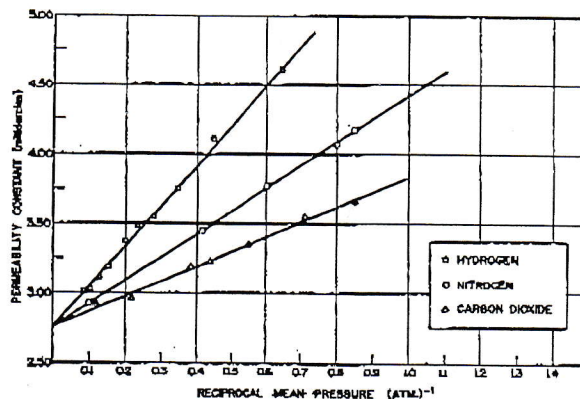
Permeability Constant of Core Sample "K" to Hydrogen, Air, and Carbon Dioxide at Different Pressures.

FIG. 4



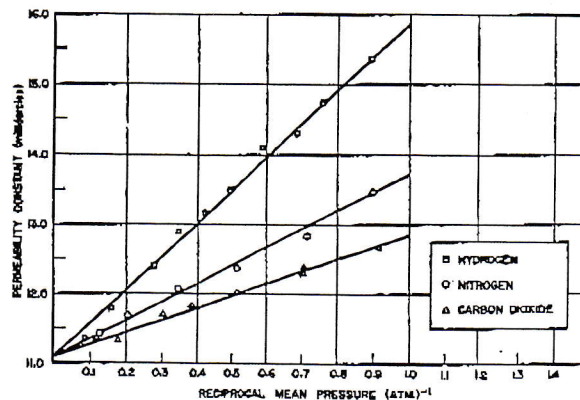
Permeability Constant of Core Sample "C" to Hydrogen, Air, and Carbon Dioxide at Different Pressures (Permeability Constant to Isooctane, 32.1 Md).

FIG. 5



Permeability Constant of Core Sample "L" to Hydrogen, Nitrogen, and Carbon Dioxide at Different Pressures (Permeability Constant to Isooctane, 2.55 Md).

FIG. 6



Permeability Constant of Core Sample "M" to Hydrogen, Nitrogen, and Carbon Dioxide at Different Pressures (Permeability Constant to Isooctane, 10.45 Md).

FIG. 7

neability as defined by *API Code No. 27*² is dependent on the nature of the fluid used. As shown, however, it is possible to determine a permeability constant which is characteristic only of the porous medium, by taking into account the phenomenon of slip; experimentally this is done simply by extrapolating the gas permeability determined at various pressures to infinite pressure.

In the oil industry the permeability of core samples

It is, therefore, evident that the discrepancies between the permeability to liquid and the permeability to gas at atmospheric pressure are in many cases—at least for formations of moderately high permeability—not of first importance for practical purposes.

For formations of low permeability, however, although the absolute discrepancies may be small, the percentage differences are considerable (see Table 9),

TABLE 9

Sample	Gas	Permeability Constant for Gas at:		Permeability Constant for Isooctane (Millidarcys)
		Atmospheric Pressure (Millidarcys)	Infinite Pressure (Millidarcys)	
Leena glass filter G, A.....	Air	4.09	2.34	2.36
Leena glass filter G, No. 3.....	Air	5.04	2.13	2.13
Leena glass filter G, No. 5.....	Air	9.89	4.36	4.36
Leena glass filter G.....	Air	97.6	"	85.6
Core sample A	Air	28.2	23.6	23.66
Core sample B	Air	28.0	"	24.4
Core sample F	Air	195	170	170
Core sample D	Air	102	"	92.3
Core sample E	Air	124.1	106.5	"
Core sample G	Air	1,096	1,054	"
Core sample H	Air	1,406	1,347	1,353
Core sample K	Hydrogen	8.0	5.9	"
Core sample K	Air	7.2	5.9	"
Core sample K	Carbon dioxide	6.9	5.9	"
Core sample C	Hydrogen	50	32	32.1
Core sample C	Air	45	32	32.1
Core sample C	Carbon dioxide	42	32	32.1
Core sample L	Hydrogen	5.64	2.75	2.55
Core sample L	Nitrogen	4.41	2.75	2.55
Core sample L	Carbon dioxide	3.84	2.75	2.55
Core sample M	Hydrogen	15.92	11.10	10.45
Core sample M	Nitrogen	13.65	11.10	10.45
Core sample M	Carbon dioxide	12.83	11.10	10.45
Core sample N	Hydrogen	20.8	14.76	14.68
Core sample N	Nitrogen	18.75	14.76	14.68
Core sample O	Hydrogen	44.70	35.50	36.20
Core sample O	Nitrogen	41.65	35.50	36.20
Core sample P	Nitrogen	68.9	60.2	61.2
Core sample R	Nitrogen	182.3	166.6	166.1
Core sample S	Nitrogen	223.0	204.3	190.7

^a The apparent permeability has not been extrapolated to infinite pressure, as only measurements at reduced pressures were made.

^b The permeability to liquid has not been determined.

is measured to establish geologic correlation and for quantitative calculation of the production rate of producing strata. In these calculations several approximations have to be made, so that it is not necessary to know the permeability with a high degree of accuracy. In addition, an oil sand in general is not homogeneous with respect to permeability, so that it is necessary to average the values for a given productive interval; and, furthermore, the permeabilities as measured in the laboratory on samples which have been extracted and dried may not represent the formation as it exists underground, because of the shrinkage of clays or other hydrated materials.

so that in these cases it is advisable to take into account the phenomena discussed hereinbefore.

VII. APPARATUS AND PROCEDURE

A. The Determination of the Permeability to Liquids

In these determinations the following points require attention:

1. The purity of the liquid: Suspended and colloidal contaminations in particular may clog the sample, and thus cause a reduction in permeability.

2. Saturation of the porous medium: During the determinations all pores should be saturated completely with liquid. Gas bubbles, if occluded in the pores, would cause the permeability of the sample to be too low.

The apparatus for the determination of the permeability with liquids is illustrated in Fig. 8.

a_1 and a_2 are the porous samples to be measured. They are mounted between two glass caps provided with normal ground joints, so that they need not be remounted when placed in another apparatus.

Glass filters were fused direct between two ground joints.

By placing two samples on top of each other, as indicated in Fig. 8, the samples can be examined simultaneously.

b_1 and b_2 serve as manometers. When small differences in pressure have to be determined, the liquid used for the permeability determination also serves as manometer liquid. (The manometer tubes are longer than indicated in the drawing.) When the pressure differences are great, the manometer tubes are filled with mercury.

c is a glass filter serving as an additional device to remove suspended particles from the liquid before the liquid passes through the samples a_1 and a_2 .

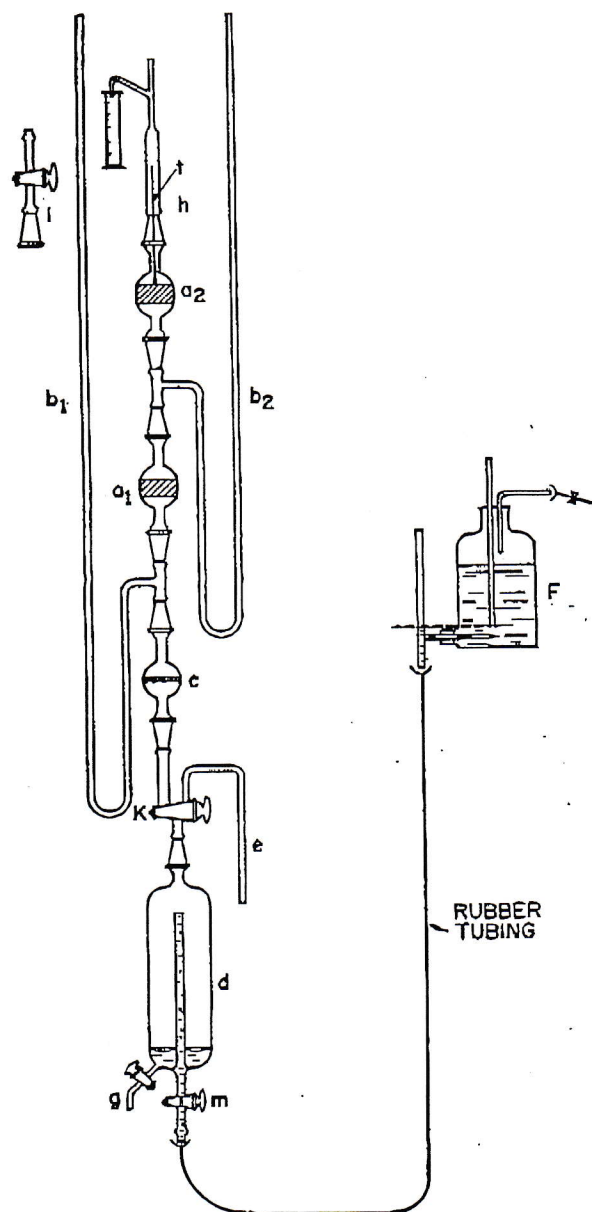
d is a reservoir of about 700-ml capacity, from which the liquid is displaced by mercury. The mercury passes from the Mariotte flask f through a central overflow into reservoir d under the influence of a constant difference in level. By means of cock k with a double bore, d can be connected at will with either the rest of the apparatus or with the side tube e .

Before the determination is started, the apparatus should be filled with the liquid and the core samples saturated. To this end, flask f , the connecting tube, and part of the central tube in d first are filled with mercury—care being taken that no air remains trapped below cock m . Vessel d then is evacuated via side tube e with a rotating oil-vacuum pump. When a satisfactory vacuum, e.g., 0.5 mm of mercury, has been obtained, side tube g is immersed in a vessel containing the freshly distilled liquid to be used. The cock in tube g is opened slightly, so that the liquid is sucked in slowly. Volatile liquids boil violently upon entering reservoir d if the pressure there is appreciably lower than their vapor pressure. The turbulences of the liquid, as it leaves the cock bore, prevent retardation in boiling. The liquid thus almost completely is freed from dissolved gases while flowing into the reservoir d , which is of great importance for the saturation of the samples.

The samples and the manometers meanwhile are evacuated through tube i , which has been substituted for the overflow h by means of a mercury diffusion pump. For this purpose the two manometers should be closed at the ends. With samples of low permeability, time frequently can be saved by simultaneously evacuating the lower manometer as well.

As soon as the pressure is about 0.01 mm mercury (measured with a McLeod manometer) cock k is turned

so as to provide a connection between vessel d and the evacuated part of the apparatus. Under the influence of the atmospheric pressure, plus the hydrostatic pressure of the mercury, the liquid passes into the ap-



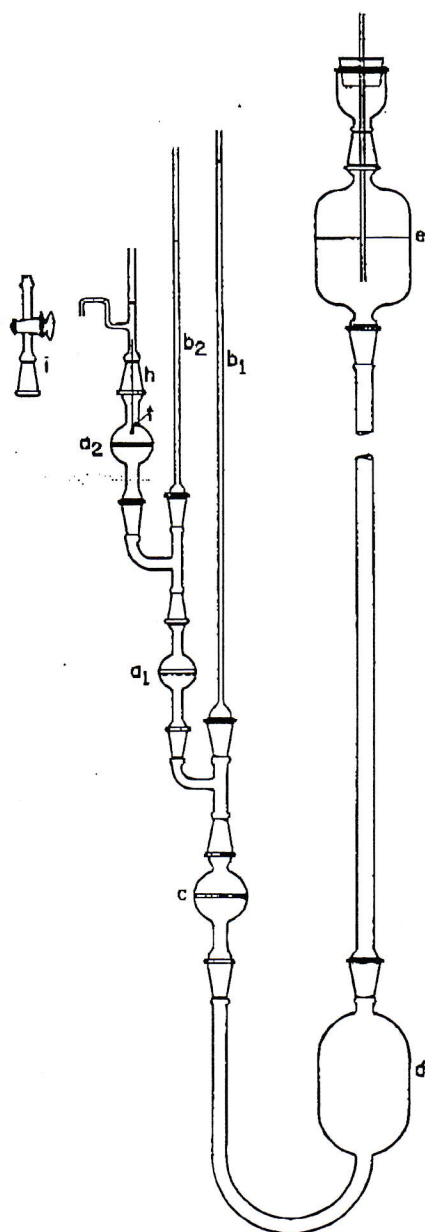
Apparatus for the Determination of the Liquid Permeability of Glass Filters and Core Samples.

FIG. 8

paratus—filling it completely. By continuous pumping during this filling operation, the last remnant of gas is displaced by the liquid vapor, while traces of gas, if liberated from the liquid, are removed as well. If no

gas bubbles are left under the samples, the pores may be assumed to be free from gas and the saturation to be complete.

Tube *i* then is replaced by overflow *h* again, after



Apparatus for the Determination of the Liquid Permeability of Glass Filters and Core Samples.

FIG. 9

thermometer *t* has been inserted for reading the liquid temperature. The apparatus is now ready for use.

The liquid then is passed through the apparatus by maintaining a constant difference in level of the mer-

cury. The difference in pressure across the sample is calculated from the levels in the manometers and in the ascending tube of *h*, after these levels have become constant. The rate of flow is measured by collecting the outflowing liquid during a measured time in a calibrated cylinder, or in a tared weighing bottle at low rates of flow.

By varying the height of flask *f*, it is possible to determine the permeability at various differences in pressure. By continuing the determinations for some time, it can be ascertained whether the permeability remains constant and whether the pores do not get clogged by contaminations.

In some cases a modified apparatus (Fig. 9) was employed. This was done if the use of mercury had to be avoided, e.g., because there was a risk of the liquid becoming contaminated when in contact with mercury. To prevent contamination by cock grease, no cocks are allowed to come in direct contact with the liquids.

*a*₁ and *a*₂ are again the glass filters (or core samples) to be measured, *b*₁ and *b*₂ the manometer tubes, and *c* an extra filter.

Prior to the determination, the reservoir *d* is filled with liquid which has been deaerated as well as possible. The apparatus then is evacuated—which takes place simultaneously through tube *i* (which is substituted for *h* during the determination), manometer tubes *b*₁ and *b*₂, and the second supply flask *c*. When the air in the samples virtually has been dispelled by liquid vapor, air is admitted into *e*; the liquid rises through *c* and fills the whole apparatus above this, so that the filters become saturated fully. If deaeration happens to be incomplete, the liquid boils when passing through *c*, so that the last traces of gas in the liquid practically are removed completely and pumped off through manometer tube *b*₁. Before the determinations, the supply flask *e* is filled also, owing to which sufficient difference in level is obtained to allow of a permeability measurement—which is carried out as described hereinbefore.

Organic liquids presented no difficulties in the determinations with glass filters—provided contamination of the freshly distilled liquids, by dust particles or contact with rubber and the like, was prevented. With water, however, difficulties were encountered, inasmuch as the permeability dropped during the determination, even if freshly distilled water was used. This was due mainly to mechanical clogging of the filters, as appeared from experiments, in which two G. filters had been placed after each other. In these experiments, made when the apparatus had not yet been provided with an extra filter (*c*, see Fig. 8 and 9), the first filter decreased much more in permeability than the second.

As contaminations causing mechanical clogging could not be supposed to be volatile and pass over during distillation, it was supposed that the troubles were caused by microorganisms developing in the water after distillation. This difficulty could be eliminated partly by sterilizing the water by adding either formaline or oxidized silver wires, and finally was eliminated entirely by the addition of the extra filter *c*.

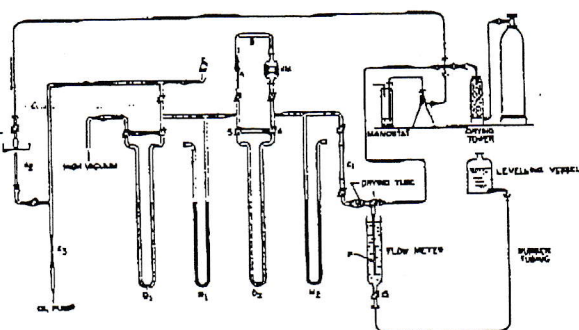
B. The Determination of the Permeability to Gases at Different Pressures

As it was desirable for our investigation to apply lower as well as higher pressures, two apparatus were constructed—one for determinations at pressures below, and one for determinations at pressures above 1 atmosphere.

The apparatus for the gas-permeability determination at low pressures is illustrated schematically in Fig. 10.

The low-pressure part is wholly made of glass, in order to prevent possible errors due to gas liberated by rubber connections.

As in the apparatus for liquid-permeability determinations, the glass filters were provided on either side with a ground joint. Core samples were fixed with Pebe cement between two hemispherical glass caps, each provided with a ground joint, as schematically



Permeability Apparatus for Low Pressures.

FIG. 10

indicated in Fig. 10 (KM). Either the glass filter or the core sample thus could be mounted by means of the ground joints. Tube B, which has two rectangular bends, was slipped over the slightly thinner tube A, and this connection was sealed with picein.

The pressure before and after the core sample can be measured direct by means of the open mercury manometers M_1 and M_2 . If the difference in pressure is slight, it can be measured more accurately with the oil manometer O_1 , which can be switched off (by means of threeway cocks 4 and 5) at larger differences in pressure.

If the pressures are of the order of a few centimeters of mercury, the pressure after the core sample can be measured by means of the oil manometer O_2 , the second limb of which is connected with the high vacuum of a mercury diffusion pump. This manometer virtually works as a closed manometer. If necessary, a correction can be made for the pressure in the "closed" limb, as the vacuum of the diffusion pump can be measured with a McLeod manometer.

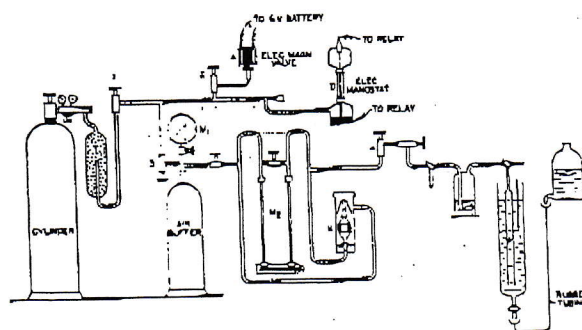
The gas is drawn through the apparatus by a rotating oil-vacuum pump.

The gas current enters the apparatus through a

calcium-chloride tube. This is generally necessary to insure reproducible results, as most core samples are more or less hygroscopic, and the permeability may change with humidity (swelling of clay particles).

The rate of flow of the gas and the pressures prevailing in the apparatus are adjusted by three exchangeable capillaries c_1 , c_2 , and c_3 . By means of these capillaries it is possible to attain any desired pressure distribution without affecting the vacuum in the pump, which serves simultaneously as pre-vacuum of the mercury diffusion pump.

To measure the rate of flow of the gas when a stationary condition has established itself, the graduated pipette P is connected by means of the threeway cock 1, and the air from the pipette is sucked through the calcium-chloride tube into the apparatus. By means of the leveling vessel and cock 13 the level outside the pipette is kept flush with that inside—so that the air,



Permeability Apparatus for High Pressures.

FIG. 11

in being sucked in, maintains a constant pressure of 1 atmosphere, and the stationary condition is not disturbed. The time required for a given volume of gas to be drawn into the apparatus is measured. As this gas is saturated with water vapor, retained in the calcium-chloride tube, a correction for this has to be made in the volume.

If the determinations are not made with air, but with hydrogen or carbon dioxide, the bomb containing this gas is connected as indicated in the drawing. The pressure is kept constant at 1 atmosphere by means of a manostat.

Fig. 11 is a schematical representation of the apparatus for gas-permeability determinations at pressures above 1 atmosphere. The sample can be mounted in this apparatus by means of the same glass hemispheres with ground joints as are used in the apparatus for low pressures and for liquid (isooctane). The sample, therefore, such as it is can be interchanged between the different apparatus. The core holder K is made of metal, and the sample is placed in it in such a way that the pressure inside and outside the glass wall is the same (apart from the slight pressure gradient in the sample), so that there is no risk of

breakage owing to the high pressure. The pressure before the sample is read on a precision manometer of the Bourdon type M; the pressure difference, across the sample on a differential manometer M₁, with dibutylphthalate as manometer liquid. The readings of the latter manometer have to be corrected for the specific gravity of the air, which, at 20 atmospheres, is no longer negligible with respect to that of the manometer liquid. The gas current is measured by conducting it into the graduated pipette P, the level inside and outside the pipette again being kept flush. The gas current is regulated by means of needle valve 4. The pressure before the sample is kept constant by means of an electromagnetic manostat. The pressure regulator consists of two chambers, connected by a glass capillary reaching close to the bottom of the lower chamber, which latter contains mercury. The top chamber is filled with air of a slightly lower pressure than that to which the apparatus is to be adjusted. When the pressure in the apparatus increases, the mercury in the capillary tube of the regulator rises until electric contact is made with an iron needle, which passes through an insulation in the wall to the middle of the capillary. A relay then is closed, and an electric current flows through the windings of the electromagnetic valve A; this opens, and gas escapes until the pressure in the apparatus drops sufficiently to interrupt again the electric contact in the regulator. The air buffer serves to level out possible fluctuations. By means of needle valves 1, 2, and 3 the manostat can be adjusted so that the pressure fluctuations in manometer M₁ amount to only a few hundredths of a centimeter of water pressure. The gas passes from a bomb to a drying bomb T containing calcium chloride. With this apparatus, permeability determinations were made at pressures up to about 20 atmospheres.

Acknowledgment

The work described in this article was carried out during 1938 and 1939, first in the laboratories of the Bataafsche Petroleum Maatschappij, Amsterdam, The Netherlands, and then in those of Shell Development Company, Emeryville, Calif., in collaboration with Dr. W. Adriani, Dr. J. Beintema, and Dr. J. van Heiningen, to whom the author is greatly indebted. Unfortunately this paper had to be prepared without their valuable advice.

The author wishes to express his sincere thanks to Dr. M. D. Taylor for several valuable discussions and help in the preparation of this paper.

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DISCUSSION

Morris Muskat (Gulf Research and Development Company, Pittsburgh, Pa.) (written): Dr. Klinkenberg's fine paper is so thorough and complete in its report of experimental data and discussion as to leave little room for additional comment.

As Dr. Klinkenberg points out, the theory of slip in the flow of gases through capillaries has been available for some 65 years. Yet those of us who have been working on permeability measurements for the last 10 years must accept with embarrassment the responsibility for having overlooked the basic applications of this theory to the flow of gases through porous media. Occasional discrepancies between permeability measurements with gases and liquids undoubtedly have been observed by all experimenters. However, it seems that the line of least resistance has been followed heretofore in attributing these discrepancies merely to experimental errors. It has remained for Dr. Klinkenberg to trace them down and to prove without question that they are an expression of a real physical phenomenon, and to expose in addition the quantitative features of the effect in such a way that it can be controlled and interpreted.

Perhaps the only excuse that can be given for previous failure to recognize the phenomenon of slip is that, if the mean free paths as established by other methods were used and the effective pore radii generally accepted as characterizing reservoir sands were introduced in the theoretical formulas for flow with slip, the correction term would have been *a priori* predicted as being far too small to play any rôle in the measurements. Of course, the basic reason for the apparently anomalous importance of the slip is the tremendous surface area in a sand as compared to that in a capillary tube. In any case, although for this reason one possibly may condone the failure of previous experimenters to find the explanation and trace down the discrepancies between the gas and liquid permeability, at the same time it makes all the more commendable Dr. Klinkenberg's perseverance and boldness in going ahead with his apparently over-optimistic working hypothesis and ultimately showing that it is, after all, well founded and significant.

A particularly valuable feature of Dr. Klinkenberg's work is that it shows how, by an extrapolation of the air-permeability data for low pressures to that for an infinitely high mean pressure, a permeability identical

with that measured with liquids will be obtained. Thus there is no need to feel that the whole method of permeability measurements with gases must be discarded. Rather it is necessary only that the measurements be carried out for several values of the mean pressure and the results extrapolated to infinite mean pressures. All the inherent advantages of gas measurements thus can be retained, and one does not have to return to the disadvantages and complexities of direct liquid measurements.

Although the reviewer has been among those who insistently have opposed the expression of sand permeability in terms of effective pore radius from the point of view of applications to the oil industry, it must be recognized that in other industrial problems the determination of the effective pore dimensions is of great value in itself. It, therefore, would be of interest to compare the effective radii corresponding to the liquid permeability and that involved in the slip phenomenon as given by the constant b . A simple check shows that bK is not at all constant, as it should be if the above effective pore radii were the same and the pore-size distribution were not of importance. However, a more detailed study of b and its variations may shed light upon the geometry of porous materials, such as might be difficult to obtain in other ways.

M. C. Leverett (Humble Oil and Refining Company, Houston, Texas) (written): This paper is an exceptional presentation of evidently painstaking experimental work, which demonstrates that in the precise measurement of permeabilities with gases the phenomenon of slip must be taken into account. Inasmuch as the importance of this phenomenon has not been appreciated previously, this paper is a valuable contribution.

Particular attention should be given the author's remark that the errors in permeability measurements with gases due to slip are "in many cases—at least for formations of moderately high permeability—not of the first importance for practical purposes." Such errors well may be significant in laboratory studies of the behavior of fluids in sands. However, in most of the fundamental studies of this type so far made the sands used have had permeabilities so high as to make slip corrections negligible.

W. S. Walls (Phillips Petroleum Company, Bartlesville, Okla.) (written): Mr. Klinkenberg's excellent paper is of interest to the general understanding and clarification of the fundamentals of flow of liquids and gases through porous media. He is to be complimented on his careful investigation of this problem.

Discrepancies between the permeabilities to air and those to water and other liquids have been observed in many core laboratories; however, the magnitude of the discrepancy encountered under routine testing conditions has not been considered sufficiently large to affect seriously the practical application of permeability data for well-completion purposes or reservoir studies.

The effects of capillary size and pressure on the flow of gases have been discussed in connection with the kinetic theory of gases and liquids in texts on physical chemistry, such as Herzfeld and Smallwood's discussion in Taylor's *Treatise on Physical Chemistry*, D. Van Nostrand Company, Inc., New York, N. Y., but Mr. Klinkenberg has clarified the matter by obtaining experimental data for actual core samples.

Although the flow of gases at low pressures may be considered to be in the region of laminar or viscous flow, some differences in the usual concepts of laminar flow are encountered in this region. At low pressures the mean free path of the gas molecules becomes large with respect to the size of the pores, and the phenomenon of slippage occurs. In some respects the usual concept of laminar or viscous flow is altered when slippage is encountered. Attempts to measure the viscosity of gases under conditions when slippage occurs will give values dependent upon the capillary size and the pressure. Viscosities measured under such conditions usually are referred to as apparent viscosities; as they are not the true viscosity of the gas, but decrease with decrease in pressure, and are dependent upon the size of the opening through which the gas is passed. Under such conditions it seems logical that Poiseuille's and Darcy's laws should not be expected to be obeyed rigorously. At high pressures the mean free path of the gas molecules becomes small with respect to the size of the pores, and slippage is not encountered. This condition is in the region of ordinary viscous flow, and Darcy's law is obeyed.

In the routine testing of cores for permeability, the deviation due to slippage is usually small when air is employed, because the mean pressures commonly used are approximately 1 atmosphere for samples having high permeability and from 1.5 to 3 atmospheres for samples having low permeability. When a Soxhlet extractor is used for extraction of the oil from the core sample in routine work, it is probable that the oil residues are not removed so thoroughly from the sands having low permeability, which will compensate partly for the divergence in permeability to air. The speed and simplicity of determining permeability to air are very much in favor of its use in routine core analysis.

Mr. Klinkenberg's paper is very useful from the standpoint of clarifying our fundamental concepts of fluid flow, but will not affect seriously the present status of core-analysis methods.

George H. Fancher (University of Texas, Austin, Texas) (written): Mr. Klinkenberg's paper is a valuable contribution to our knowledge of what is probably the most important single physical property of oil and gas sands, viz., the permeability. Despite the considerable amount of research which has been conducted in this field during the past decade and the impressive array of data and information concerning permeability which has been accumulated, and the further fact that the Klinkenberg phenomena have been known a long time, the work is new, timely, and significant—especially

from the theoretical point of view. There is no question that it has been proved that, when the permeability of a porous medium is measured with gas, allowance and a correction for the fact that the test fluid is a gas rather than a liquid must be made in order to determine the true permeability of the medium. Nowhere in the literature is there more convincing proof of the fact that permeability is inherently a property of the medium when liquids are used to measure permeability. The theory for flow of gas is supported by convincing data, although there remains opportunity for further work and fact finding in tying the theory down. Complete test data would be useful to all workers in the field in order to hasten progress, and it is hoped that Mr. Klinkenberg will find it possible to supply these data.

It is interesting to recall, in the light of this new information, that in comparing the permeability of a sand measured with a gas to that with a liquid, some investigators have reported better agreement when the Weymouth mean pressure was used rather than the arithmetic mean pressure demanded by theory. When it is recalled further that the Weymouth mean always exceeds the arithmetic by an appreciable amount, the matter is clarified, especially for test pieces of low permeability over which a pressure drop of 100 or 150 psi may be used in experimentation. Of course, no physical nor theoretic justification of the Weymouth mean can be made for this work, and its use is not necessary nor warranted if the Klinkenberg method be followed.

Practically speaking, the permeability of a sand in excess of, shall we say, 25 millidarcys, may be evaluated in the usual simple way, using air as a test fluid, with satisfactory overall accuracy. Considering the fact that we usually are interested commercially in permeability higher than 25 millidarcys for practical reasons and that the small piece being tested is not thoroughly representative of the entire sand body, it is believed that Mr. Klinkenberg's findings will not affect routine permeability measurements except in the case of tight sands (low permeability), and no

serious difficulties will result in following usual methods of testing permeability in the majority of cases.

M. D. Taylor (Shell Development Company, Emeryville, Calif.) (written):* The universally favorable tone of the reviews and comments is, of course, most gratifying. In reply, I should like to agree with the comment made by several of the reviewers, viz., that the effect of slip is usually too small to be of practical importance in field calculations, and to repeat that other effects incidental to the recovery of representative core samples and their preparation for measurement easily may introduce more serious errors than neglect of the effect discussed in this paper. The adoption of a limiting permeability above which the effect of slip can be neglected in practical calculations, such as a permeability of 25 millidarcys as suggested by Mr. Fancher, does not appear to be particularly attractive. A plot of the data presented in Table 9 for core samples measured with air or nitrogen shows that neglecting the effect of slip for core samples of 10 millidarcys will cause, on the average, an error of about 25 per cent (based on the value measured at atmospheric pressure); for a sample of 25 md the average error would be about 17 per cent; for a 50-md sample it is about 13 per cent; for a 100-md sample about 11 per cent, and for 1,000-md sample about 5 per cent. It appears, therefore, that as the permeability increases, the error caused by neglecting slip decreases gradually, and there is no limiting permeability beyond which the error drops rapidly.

The use of variations observed in the quantity b to obtain information on the geometry of a porous material, as suggested by Mr. Muskat, has been the subject of considerable investigation, but no definite conclusions have been reached. It appears that the significant factors are; 1, the radii of the capillaries at their narrowest parts; 2, the rate with which these radii increase; 3, the length of the narrow portions; and, 4, the distribution of capillary pore sizes in the sample as a whole.

* Author's closure prepared by M. D. Taylor due to the author's absence in army service.